Solutions to Sample problems for Exam 1 for Math 233

Please report any mistake or typos to your instructor.

1. (a)$\langle\, 2, -6, 4 \,\rangle$ (b)$\langle\, 14, 3, -17 \,\rangle$ (c)$\sqrt{26}$ (d)$\sqrt{38}$ (e)$2\sqrt{14}$ (f)-4
   (g)$\langle\, -18, -18, -18 \,\rangle$ (h)$\langle\, -(8/13), 6/13, 2/13 \,\rangle$ (i) $\langle\, 4/19, 6/19, -(10/19) \,\rangle$

2. $\cos\theta = 9/\sqrt{870}$, so $\theta \simeq 72°$

3. $\vec{\mathbf{m}} = \langle\, -(20/9), 10/9, 20/9 \,\rangle$ and $\vec{\mathbf{n}} = \langle\, 38/9, 26/9, 25/9 \,\rangle$.

4. Area$= 9\sqrt{2}$

5. Area$= 3$

6. $n = \pm\frac{1}{\sqrt{86}}(7, -1, 6)$

7. Parametric: $x = 4 - t,\ y = 5 - 3t,\ z = 6 - 5t$, symmetric: $x - 4 = \frac{y-5}{3} = \frac{z-6}{5}$

8. parametric: $\ell(t) = (1 - 13t, -13t, 1)$, symmetric: $x - y = 1,\ z = 1$

9. $2x - y - z + 3 = 0$

10. $3x - 5y + z - 1 = 0$

11. $3x - y + z = 0$

12. No. Use dot product, cross product, or equations to justify. For example, the cross product $(1, 0, 2) \times (0, 1, 1) \neq 0$.

13. No. Points are coplanar if and only if the scalar triple product of the 3 vectors obtained from the 4 points is zero. The scalar triple product is 2 (or -2) and hence points are no coplanar.

14. $2x + 4y - z + 11 = 0$

15. $1/\sqrt{11}$

16. Hyperbolic paraboloid (saddle), which passes through the origin, rises along the x–axis, and falls along the y–axis.

17. Hyperboloid of two sheets with circular traces of intersection with planes $x = k$ (for $k \geq 2$).

18. $(r, \theta, z) = (2, \pi/6, 2\sqrt{3})$ and $(\rho, \theta, \phi) = (4, \pi/6, \pi/6)$

19. The surface is a half cone with cone angle $\pi/3$. In rectangular co-ordinate: $z = \sqrt{3}\sqrt{x^2 + y^2}$ In cylindrical co-ordinates: $z = \sqrt{3}\ r$