## Sample problems for Exam 1 for Math 233

This sample exam has many more questions than the actual exam will have.

1. Let $\overrightarrow{\mathbf{a}}=\langle 4,-3,-1\rangle$ and $\overrightarrow{\mathbf{b}}=\langle-2,-3,5\rangle$. Find $\quad$ (a) $\overrightarrow{\mathbf{a}}+\overrightarrow{\mathbf{b}} \quad$ (b) $2 \overrightarrow{\mathbf{a}}-3 \overrightarrow{\mathbf{b}}$
(c) $|\overrightarrow{\mathbf{a}}|$
(d) $|\overrightarrow{\mathbf{b}}|$
(e) $|\vec{a}+\vec{b}|$
(f) $\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}}$
(g) $\overrightarrow{\mathbf{a}} \times \overrightarrow{\mathbf{b}}$
(h) $\operatorname{proj}_{\overrightarrow{\mathbf{a}}} \overrightarrow{\mathbf{b}}$
(i) $\operatorname{proj}_{\overrightarrow{\mathrm{b}}} \overrightarrow{\mathbf{a}}$
2. Find the angle between the vectors $\overrightarrow{\mathbf{a}}=-4 \overrightarrow{\mathbf{i}}+2 \overrightarrow{\mathbf{j}}+3 \overrightarrow{\mathbf{k}}$ and $\overrightarrow{\mathbf{b}}=2 \overrightarrow{\mathbf{i}}+\overrightarrow{\mathbf{j}}+5 \overrightarrow{\mathbf{k}}$.

Find the angle that $\overrightarrow{\mathbf{a}}$ makes with the $z$-axis. (Note this is its $\phi$ in spherical coordinates.)
3. Express the vector $\overrightarrow{\mathbf{u}}=\langle 2,4,5\rangle$ as a sum of vector $\overrightarrow{\mathbf{m}}$ parallel to $\overrightarrow{\mathbf{v}}=\langle 2,-1,-2\rangle$, and a vector $\overrightarrow{\mathbf{n}}$ perpendicular to $\overrightarrow{\mathbf{v}}$.
4. Find the area of the triangle with vertices $(1,2,3),(3,1,5)$ and $(4,5,6)$.
5. Find the area of the parallelogram with vertices $(-1,2,0),(1,0,1),(-2,0,0)$ and $(0,-2,1)$.
6. Find unit vectors perpendicular to the plane passing through the points $(1,3,5),(3,-1,2)$ and $(4,0,1)$.
7. Find the parametric and symmetric equations of the line passing through the points $(4,5,6)$ and $(3,2,1)$.
8. Find the parametric and symmetric equations of the line at the intersection of the planes, $3 x-3 y-7 z=-4$ and $x-y+2 z=3$.
9. Find the equation of the plane passing through the points $(1,3,2),(0,3,0)$ and $(2,4,3)$.
10. Find the equation of a plane passing through point $(2,1,0)$ and containing the line $\frac{x-3}{2}=y=\frac{-8-z}{1}$.
11. Find the equation of a plane perpendicular to the line $x=2+3 t, y=-t, z=-1+t$ and passing through point $(0,1,-1)$.
12. Determine whether the points $(1,1,2),(0,1,0)$ and $(1,2,3)$ are collinear.
13. Determine whether the points $(0,2,1),(0,1,0),(1,1,0)$ and $(1,2,3)$ are coplanar.
14. Find the equation of the plane perpendicular to the line $x=1+2 t, y=-3+4 t$, $z=-1-t$ and passing through the point $(-1,-1,5)$.
15. Find the distance between the parallel planes $3 x+3 y-9 z=4$ and $x+y-3 z=1$.
16. Describe and sketch the surface $z=x^{2}-4 y^{2}$.
17. Describe and sketch the surface $x^{2}-y^{2}-z^{2}-4=0$.
18. Find the cylindrical and spherical coordinates for the point $P(\sqrt{3}, 1,2 \sqrt{3})$.
19. Describe and sketch the surface given in spherical coordinates by $\phi=\pi / 3$. Find the equations in cylindrical and cartesian coordinates for this surface.

