Calculus III (Math 233) Exam 3

November 24, 2008 Professor Ilya Kofman

Justify answers and show all work for full credit.

NAME: ____

Problem 1. *(14 pts)* Find the equation of the tangent plane to the surface $xy + yz + zx = 6$ at the point $(2, 0, 3)$.

Problem 2. *(14 pts)* The temperature at a point in the plane is $T(x, y) = 100 - 3x^2 - 2y^3$. A bug is at the point $(1, -1)$.

(a) Compute $\nabla T(1, -1)$.

(b) Find the rate of change of temperature in the direction of $\vec{v} = \langle 3, -4 \rangle$.

(c) Find a direction in which the bug should move to NOT change its temperature.

Problem 3. *(8 pts)* Suppose the plane $z = x - 2y - 3$ is tangent to the graph of $z = f(x, y)$ at $P(1, -2)$.

(a) Find $f(1, -2)$, $f_x(1, -2)$, $f_y(1, -2)$.

(b) Find the direction of maximum increase for the function f at the point P.

Problem 4. *(14 pts)* A rectangular box has length, width and height, respectively, $20\,cm \times 10\,cm \times 10\,cm$. Use differentials to estimate the maximum error in measuring the volume of the box if the error in measuring each dimension is $\pm 0.11\,cm$.

Problem 5. *(14 pts)* Let $f(x, y, z) = x + y^2 z$ and $x = 3s^2 + 2t$, $\quad y = 3s - 2t^2$ and $z = s^2 - t^2$. Compute $\dfrac{\partial f}{\partial s}(2, -2)$ and $\dfrac{\partial f}{\partial t}(2, -2)$.

Problem 6. *(22 pts)* Let $f(x, y) = 2x^2 + y^2 - 4y + 3$.

(a) Find critical points of f on the region $x^2 + y^2 < 9$.

(b) Find the exterme values on the boundary $x^2 + y^2 = 9$ using Lagrange Multipliers.

(c) Find the exterme values of f on $x^2 + y^2 \leq 9$ using the above information.

Problem 7. *(14 pts)* Find all the critical points of $f(x, y) = x^3 + y^3 - 3xy + 4$, and classify them using the Second Derivative Test.