Exam 2 - Mth 233 10/22/2008 Solutions

① $\vec{r}_0 = (2,1,-1)$, $\vec{v}_0 = (3,-4,5)$, $\vec{a} = \vec{r}''(t) = (6t, 12t^2, -6)$

$\vec{r}'(t) = \int (6t, 12t^2, -6)\,dt = (3t^2, 4t^3, -6t) + (3,-4,5)$

$\vec{r}(t) = \int (3t^2+3,\ 4t^3-4,\ -6t+5)\,dt = (t^3+3t,\ t^4-4t,\ -3t^2+5t) + (2,1,-1)$

$\vec{r}(2) = (8+6+2,\ 16-8+1,\ -12+10-1) = \underline{\underline{(16,9,-3)}}$

② $\vec{r}_0 = (0,4)$ $\vec{v}_0 = (3,0)$ $\vec{a} = (0,-32) \Rightarrow \vec{v}(t) = \int (0,-32)\,dt + (3,0) = (3,-32t)$

$\vec{r}(t) = -16t^2\vec{j} + 3t\vec{i} + 4\vec{j} = (3t,\ 4-16t^2)$

Ball lands when $16t^2 = 4 \Rightarrow t^2 = \frac{1}{4} \Rightarrow t = \frac{1}{2}$ sec

Its displacement at $t = \frac{1}{2}$ is $3(\frac{1}{2}) = \underline{\underline{3/2 \text{ ft}}}$

③ $\vec{r}(t) = (e^t, \sqrt{2}t, e^{-t}) \Rightarrow \vec{r}'(t) = (e^t, \sqrt{2}, -e^{-t})$

a) $v = \|\vec{r}'(t)\| = \sqrt{e^{2t} + 2 + e^{-2t}} = \sqrt{(e^t+e^{-t})^2} = \underline{\underline{e^t + e^{-t}}}$

b) $\vec{T}(t) = \dfrac{\vec{r}'(t)}{\|\vec{r}'(t)\|} = \underline{\underline{\dfrac{1}{e^t+e^{-t}}(e^t, \sqrt{2}, -e^{-t})}}$

c) $\vec{a}(t) = \vec{r}''(t) = (e^t, 0, e^{-t}) \Rightarrow a_T = \vec{a}\cdot\vec{T} = \dfrac{1}{e^t+e^{-t}}(e^{2t} - e^{-2t})$

(OR $a_T = v'(t) = \frac{d}{dt}(e^t+e^{-t}) = e^t - e^{-t}$.) $= \dfrac{(e^t+e^{-t})(e^t-e^{-t})}{e^t+e^{-t}} = \underline{\underline{e^t - e^{-t}}}$

d) $a_N = \sqrt{\|\vec{a}\|^2 - a_T^2} = \sqrt{(e^{2t}+e^{-2t}) - (e^t-e^{-t})^2}$

$= \sqrt{e^{2t}+e^{-2t} - (e^{2t} - 2 + e^{-2t})} = \underline{\underline{\sqrt{2}}}$

e) $s = \int_1^3 \|r'(t)\|\,dt = \int_1^3 (e^t + e^{-t})\,dt = \left[e^t - e^{-t}\right|_1^3 = \underline{\underline{e^3 - e^{-3} - e + e^{-1}}}$

④ a) $\vec{r}(t) = (4\cos t, 4\sin t, \frac{15}{6\pi}t)$, $0 \le t \le 6\pi$.

b) $s = \int_0^{6\pi} \|r'(t)\|\,dt = \int_0^{6\pi} \sqrt{4^2 + (15/6\pi)^2}\,dt = \underline{\underline{6\pi\sqrt{16 + (15/6\pi)^2}}}$

⑤ $\vec{r}(t)\cdot\vec{r}(t) = c^2 \Rightarrow \vec{r}\cdot\vec{r}' + \vec{r}'\cdot\vec{r} = 0 \Rightarrow 2\vec{r}\cdot\vec{r}' = 0 \Rightarrow \vec{r}\cdot\vec{r}' = 0 \Rightarrow \vec{r} \perp \vec{r}'$

⑥ $\vec{r}(t) = \vec{p} + t\vec{v} \Rightarrow T(t) = \dfrac{\vec{v}}{\|\vec{v}\|} \Rightarrow \vec{T}'(t) = 0 \Rightarrow K = 0$ for all t.

⑦ $\|r'(s)\| = 1 \Rightarrow r'(s)\cdot r'(s) = 1 \Rightarrow \vec{r}'\cdot\vec{r}'' + \vec{r}''\cdot\vec{r}' = 0 \Rightarrow 2\vec{r}'\cdot\vec{r}'' = 0 \Rightarrow \vec{r}'\cdot\vec{r}'' = 0$

$\Rightarrow \vec{v}\cdot\vec{a} = 0$