Calculus III (Math 233) Exam 2

Date: October 22, 2008 Professor Ilya Kofman

Justify answers and show all work for full credit.

NAME: _____

Problem 1. A particle starts at location $2\,\mathbf{i}+\mathbf{j}-\mathbf{k}$ with initial velocity $3\,\mathbf{i}-4\,\mathbf{j}+5\,\mathbf{k}$. Its acceleration is $\mathbf{a}(t) = 6t\,\mathbf{i} + 12t^2\,\mathbf{j} - 6\,\mathbf{k}$. Find the location of the particle at $t = 2$.

Problem 2. On a flat table 4 ft high, a ball rolls with a speed of 3 ft/sec. If the ball rolls off, how far away from the table does it land?

Problem 3. The position of a particle is $\mathbf{r}(t) = e^t\,\mathbf{i} + \sqrt{2}\,t\,\mathbf{j} + e^{-t}\,\mathbf{k}$.

(a) Show that the speed of the particle at time t is $e^t + e^{-t}$.

(b) Find the unit tangent vector $\mathbf{T}(t)$.

(c) Find the tangential component of the acceleration.

(d) Find the normal component of the acceleration.

(e) Find the total distance travelled by the particle for $1 \leq t \leq 3$.

Problem 4. A string in the shape of a helix has a height of 15 cm and makes three full rotations over a circle of radius 4 cm.

(a) Find a parametrization $\mathbf{r}(t)$ for the string.

(b) Compute the length of the string.

Problem 5. Show that if $||\mathbf{r}(t)|| = c$ then $\mathbf{r}(t) \perp \mathbf{r}\,'(t)$.

Problem 6. Show that the curvature of a straight line in space at every point is 0.

Problem 7. (**Bonus**)
Show that if $\mathbf{r}(s)$ is parametrized by arclength then $\mathbf{a}(s) \cdot \mathbf{v}(s) = 0$.