Calculus III (Math 233) Exam 1

Date: September 24, 2008 Professor Ilya Kofman

Justify answers and show all work for full credit.

NAME: _____

Problem 1. Let $\vec{\mathbf{u}} = \langle\, 4, 4, 5 \,\rangle$ and $\vec{\mathbf{v}} = \langle\, 2, -1, 1 \,\rangle$.

(a) Find a unit vector in the direction of $\vec{\mathbf{v}}$.

(b) Find $||\text{proj}_{\vec{\mathbf{v}}}\vec{\mathbf{u}}||$.

(c) Express $\vec{\mathbf{u}}$ as the sum of $\vec{\mathbf{m}} = \vec{\mathbf{u}}_{||}$ parallel to $\vec{\mathbf{v}}$, and $\vec{\mathbf{n}} = \vec{\mathbf{u}}_{\perp}$ orthogonal to $\vec{\mathbf{v}}$.

Problem 2. Consider three points $A(-2, 1, -1),\ B(1, 2, 2),\ C(1, 1, 5)$.

(a) Are the points A, B, C collinear? Justify your answer using the cross-product.

(b) Find the area of the triangle formed by A, B, C.

(c) Find the equation of the plane that contains A, B, C.

Problem 3. Consider two points $E(1, 0, 1),\ F(-3, 2, 3)$.

(a) Find a parametric equation of the line through E and F.

(b) Find the symmetric equation of the line through E and F.

(c) Find the cylindrical coordinates for E.

(d) Find the spherical coordinates for E.

Problem 4.

(a) Find the angle between the planes $x - y = 3$ and $-y + z = 1$.
(Hint: Angle between the planes is the angle between their normal vectors.)

(b) Find the equation of the plane that passes through the point $(1, 2, -1)$ and is perpendicular to the line $x - 2 = \dfrac{y+1}{2} = \dfrac{z}{4}$.

Problem 5. For each equation below, find the surface in $\mathbf{R}^3$ that matches it.

(a) _____ $x^2 + 4y^2 + 4z^2 = 16$

(b) _____ $4x^2 + y^2 + 4z^2 = 16$

(c) _____ $z = 9x^2 + 4y^2$

(d) _____ $z = 9x^2 - 4y^2$

(e) _____ $9x^2 + 4y^2 = 2z^2 + 72$

(f) _____ $9x^2 + 4z^2 = 2y^2 - 72$

(g) _____ $9x^2 + 4y^2 = 2z^2$

(h) _____ $9x^2 - 4y^2 = 72$