Sample problems for Exam 2 for Math 232 (Kofman)

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1. Evaluate the following improper integrals.

   (a) $\displaystyle\int_1^\infty \frac{1}{(2x+1)^3}\,dx$ (b) $\displaystyle\int_2^6 \frac{y}{\sqrt{y-2}}\,dt$ (c) $\int_{-\infty}^0 e^{3t}\,dt$

2. Determine whether the following series converge or diverge. Indicate which test you are using.

   (a) $\displaystyle\sum_{n=1}^\infty \frac{3^n}{5^{n-1}}$ **(b)** $\displaystyle\sum_{n=1}^\infty 2^{1-n}$

   (c) $\displaystyle\sum_{n=1}^\infty \frac{1}{n\sqrt{n}} - \frac{3}{n^3}$ **(d)** $\displaystyle\sum_{n=1}^\infty \frac{1+2^n+3^n}{5^n}$

   (e) $\displaystyle\sum_{n=3}^\infty \frac{1}{n(\ln n)^2}$ **(f)** $\displaystyle\sum_{n=1}^\infty \frac{1}{n^2+n+1}$

   (g) $\displaystyle\sum_{n=1}^\infty \frac{n^2-n}{n^4+2}$ **(h)** $\displaystyle\sum_{n=2}^\infty \frac{n}{n^3-1}$

   (i) $\displaystyle\sum_{n=1}^\infty \frac{n^2+1}{3n^3+2}$ **(j)** $\displaystyle\sum_{n=1}^\infty \frac{2n+3}{3n-5}$

   (k) $\displaystyle\sum_{n=1}^\infty \frac{(-1)^n}{4n+5}$ **(l)** $\displaystyle\sum_{n=1}^\infty (-1)^n \sin n$

   (m) $\displaystyle\sum_{n=1}^\infty \frac{(-1)^n n}{3n+2}$ **(n)** $\displaystyle\sum_{n=1}^\infty \frac{(-1)^n}{\sqrt{2^n+1}}$

   (o) $\displaystyle\sum_{n=1}^\infty \frac{\cos(n\pi)}{n^{3/2}}$ **(p)** $\displaystyle\sum_{n=1}^\infty \frac{3^n}{(n!)}$

   (q) $\displaystyle\sum_{n=1}^\infty \frac{n^3}{5^n}$ **(r)** $\displaystyle\sum_{n=1}^\infty \frac{(-5)^{2n}}{n^2 9^n}$

3. Find the radius of convergence of the following power series.

   (a) $\displaystyle\sum_{n=0}^\infty \frac{nx^n}{3^n}$ **(b)** $\displaystyle\sum_{n=0}^\infty \frac{(x-2)^n\sqrt{n}}{n^2+1}$

   (c) $\displaystyle\sum_{n=0}^\infty \frac{(-1)^n(x+3)^{2n}}{4^n}$ **(d)** $\displaystyle\sum_{n=0}^\infty \frac{(n+1)(2x+1)^n}{3^{n+1}}$

4. Find the interval of convergence of the following power series.

(**a**) $\sum_{n=0}^{\infty} \frac{(2-x)^n}{3n+1}$ (**b**) $\sum_{n=0}^{\infty} \frac{(-1)^n (x+3)^{2n}}{4^n}$

(**c**) $\sum_{n=0}^{\infty} \frac{3^n (x-2)^n}{2^n}$ (**d**) $\sum_{n=1}^{\infty} \frac{n^2 (x-1)^{3n}}{2^n}$

5. Suppose that the series $\sum_{n=1}^{\infty} c_n (x-1)^n$ converges when $x = 4$ and diverges when $x = 6$. What can be said about the convergence or divergence of the following series ?

(**a**) $\sum_{n=1}^{\infty} c_n 2^n$ (**b**) $\sum_{n=1}^{\infty} c_n (-1)^n 3^n$ (**c**) $\sum_{n=1}^{\infty} c_n (-1)^n$

6. Find a power series representations of the following functions.

(**a**) $\mathrm{f(x)} = \tan^{-1}(3\mathrm{x})$ (**b**) $\mathrm{f(x)} = \frac{\mathrm{x}^3}{(1+\mathrm{x})^2}$ (**c**) $\mathrm{f(x)} = \ln(1+\mathrm{x})$

(**d**) $\mathrm{f(x)} = \mathrm{e}^{2(\mathrm{x}-1)^2}$ (**e**) $\mathrm{f(x)} = \frac{\sin(3\mathrm{x}^2)}{\mathrm{x}^3}$ (**f**) $\mathrm{f(x)} = \int \mathrm{e}^{\mathrm{x}^2}$

7. Find Maclaurin series of the following:

(**a**) $\mathrm{f(x)} = \mathrm{e}^{\mathrm{x}}$ (**b**) $\mathrm{f(x)} = \mathrm{e}^{5\mathrm{x}}$ (**c**) $\mathrm{f(x)} = \sin 2\mathrm{x}$ (**d**) $\mathrm{f(x)} = \cos 3\mathrm{x}$

8. Find the Taylor series of the given function at the given point a.

(**a**) $\mathrm{f(x)} = \mathrm{e}^{2\mathrm{x}}, \quad \mathrm{a} = 2$ (**b**) $\mathrm{f(x)} = \frac{1}{\mathrm{x}}, \quad \mathrm{a} = -3$

(**c**) $\mathrm{f(x)} = \sqrt{1+\mathrm{x}}, \quad \mathrm{a} = 0$

9. By recognizing each of the following series as a Taylor series evaluated at a particular value of x, find the sum of each of the following convergent series.

(a) $$1 + \frac{2}{1!} + \frac{4}{2!} + \frac{8}{3!} + \cdots + \frac{2^n}{n!} + \cdots$$

(b) $$1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \cdots + \frac{(-1)^n}{(2n+1)!} + \cdots$$

(c) $$1 + \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \cdots + \frac{1}{4^n} + \cdots$$

(d) $$1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \cdots$$

10. Find the Taylor or Maclaurin polynomial for the given function, for the given degree, centered at the given point.

(a) $f(x) = \frac{1}{x^2},\ n = 4,\ c = 1.$

(b) $f(x) = \ln x,\ n = 4, = 2.$

(c) $f(x) = \sin(3x),\ n = 5,\ c = 0.$

(d) $f(x) = \sqrt{x},\ n = 4,\ c = 1.$

Test	Description	Examples and Comments
geometric series	$\sum_{n=0}^{\infty} ar^n$ converges to $a/(1-r)$ if $\|r\| < 1$ and diverges if $\|r\| \geq 1$.	$\sum_{n=0}^{\infty}(1/2)^n$ converges to 2; $\sum_{n=0}^{\infty} 2^n$ diverges.
Divergence test	If $\|a_n\|$ does not converge to 0, then $\sum a_n$ diverges.	If $a_n \to 0$, then $\sum a_n$ may converge ($\sum 1/n^2$) or it may not (the harmonic series $\sum 1/\text{n}$).
p-series	$\sum_{n=1}^{\infty} 1/n^p$ diverges if $0 \leq p \leq 1$ and converges if $p > 1$.	
integral test	Suppose $a_n = f(n) \geq 0$ and f is decreasing. If $\int_1^{\infty} f(x)dx$ converges then $\sum a_n$ converges. If $\int_1^{\infty} f(x)dx$ diverges then $\sum a_n$ diverges.	Use this test whenever $f(x)$ can easily be integrated.
comparison test	Suppose $0 \leq a_n \leq b_n$. If $\sum b_n$ converges then $\sum a_n$ also converges. If $\sum a_n$ diverges, then $\sum b_n$ also diverges.	It is not necessary that $a_n \leq b_n$ for all n, only for $n \geq N$ for some integer N; convergence or divergence of a series is not affected by the values of the first few terms.
ratio test	If $\lim\limits_{n\to\infty} \frac{\|a_{n+1}\|}{\|a_n\|} = L$ then $\sum a_n$ converges if $L < 1$ and diverges when $L > 1$.	This is often the easiest test to apply; note that if $L = 1$, then the series may either converge ($\sum 1/n^2$) or diverge ($\sum 1/n$).
root test	If $\lim\limits_{n\to\infty} \sqrt[n]{\|a_n\|} = L$ then $\sum a_n$ converges if $L < 1$ and diverges when $L > 1$.	To be used when a_n have exponents in terms of n (e.g. $\sum \frac{3^n}{n^n}$). Note that if $L = 1$, then the series may either converge ($\sum 1/n^2$) or diverge ($\sum 1/n$).
alternating series test	$\sum(-1)^n a_n$ converges if (i) $a_n \to 0$ as $n \to \infty$ and (ii) $0 \leq a_{n+1} < a_n$.	This test can only be applied when the terms are alternately positive and negative; if there are two or more positive (or negative) terms in a row, then try another test.
absolute convergence test	If $\sum \|a_n\|$ converges, then $\sum a_n$ converges.	To determine whether $\sum \|a_n\|$ converges, try any of the tests that apply to series with nonnegative terms.