Calculus II (Math 232) Quiz

December 10, 2014 — Professor Ilya Kofman

Justify answers and show all work for full credit. No graphing calculators.

NAME: Key

1. Find the surface area of revolution about the x–axis for $y = x^3$ for $0 \le x < 2$.
2. Find a path $c(t)$ that traces the line $y = 2x + 3$ from $(2, 7)$ to $(4, 11)$ for $0 \le t \le 1$.
3. Sketch the (real part of the) polar curve $r = \sqrt{\sin(2\theta)}$.
4. Find the area that is inside the curve $r = \sqrt{2}\,\sin(\theta)$ and outside the unit circle.

① $SA = \int 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$ $\qquad u = 1+9x^4$

$= \int_0^2 2\pi x^3\sqrt{1+(3x^2)^2}\,dx = 2\pi\int_0^2 x^3\sqrt{1+9x^4}\,dx$ $\qquad du = 36x^3\,dx$

$= \frac{2\pi}{36}\int_1^{145}\sqrt{u}\,du = \frac{2\pi}{36}\cdot\frac{2}{3}u^{3/2}\Big|_1^{145} = \frac{\pi}{27}\left(145^{3/2}-1\right)$

② $P(2,7) \longrightarrow Q(4,11)$ as $t: 0 \to 1$ $\qquad$ let $x(t) = 2t+2$

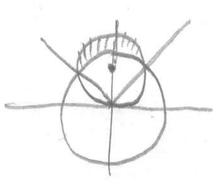
then $y(t) = 2x(t)+3 = 2(2t+2)+3 = 4t+7$

$c(t) = (2t+2,\ 4t+7)$, so then $c(0) = P$, $c(1) = Q$.

③

θ	r
0	0
$\pi/4$	1
$\pi/2$	0
π	0
$5\pi/4$	1
$3\pi/2$	0

④



$\sqrt{2}\sin\theta = 1$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

$r = \sqrt{2}\sin\theta$ is a circle of radius $\frac{\sqrt{2}}{2}$ centered at $(0, \sqrt{2}/2)$

$A = \int_{\pi/4}^{3\pi/4} \frac{1}{2}R^2 - \frac{1}{2}r^2\,d\theta$

$= \int_{\pi/4}^{3\pi/4} \frac{1}{2}\left(2\sin^2\theta - 1\right)d\theta$

$= \int_{\pi/4}^{3\pi/4} \sin^2\theta - \frac{1}{2}\,d\theta$ $\qquad \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$= \int_{\pi/4}^{3\pi/4} -\frac{1}{2}\cos 2\theta\,d\theta = -\frac{1}{4}\sin 2\theta\Big|_{\pi/4}^{3\pi/4}$

$= -\frac{1}{4}(-1-1) = \frac{1}{2}$