Calculus I (Math 231) Exam 3

Date: December 10, 2007 Professor Ilya Kofman

Justify answers and show all work for full credit. No symbolic calculators allowed.

NAME: ____

Problem 1. (1) Find the critical points, (2) Find the inflection points, (3) Find intervals where it is concave up or down, (4) Identify all relative extrema using the Second Derivative Test.

$$f(x) = 2 + 2x^2 - x^4$$

Problem 2. Sketch the graph of a differentiable function $f(x)$ with all of the following properties:

- The domain of f is $(-\infty, 2) \cup (2, \infty)$.
- $\lim_{x \to 2} f(x) = -\infty$.
- $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = \infty$.
- $f'(x) > 0$ for $x < -1$ and for $x > 2$.
- $f'(x) < 0$ for $-1 < x < 2$.
- $f''(x) > 0$ for $x < -3$ and for $x > 3$.
- $f''(x) < 0$ for $-3 < x < 2$ and for $2 < x < 3$.
- $f(-3) = 1$, $f(-1) = 3$, and $f(3) = 0$.

Label all horizontal and vertical asymptotes, local extrema, and inflection points.

Problem 3. A cylindrical can with height h and radius r will be made to hold $16\,\mathrm{cm}^3$ of oil. Find the dimensions that will minimize the metal to manufacture the can.

Problem 4. Evaluate

(a) $\displaystyle\int (t - \sin t)\ dt$

(b) $\displaystyle\int \left(-3x^5 + \sqrt[3]{x^2} + \frac{1}{\sqrt{x}}\right)\ dx$

(c) $\displaystyle\int \frac{dx}{x\sqrt{\ln x}}$

Problem 5. Evaluate the Riemann sum for $f(x) = 10 - 2x$, for $2 \le x \le 14$, with $n = 4$ subintervals, taking the sample points to be the right endpoints.

Problem 6. Evaluate

(a) $\displaystyle\int_{-1}^{2} (8x^3 - 2)\ dx$

(b) $\displaystyle\int_{1/2}^{1} (2t - 1)^{25}\ dt$

(c) $\displaystyle\int_{0}^{1} xe^{-x^2}\ dx$

Problem 7. Archimedes showed that the area of a parabolic arch is equal to $\frac{2}{3}$ of the product of its base and height. Verify this formula for the parabolic arch bounded by $y = 9 - x^2$ and the x–axis.