Math 231 Exam 3 Solutions 12/10/2007

① $f(x) = 2 + 2x^2 - x^4$

$f'(x) = 4x - 4x^3 = 4x(1-x)(1+x) \overset{\text{set}}{=} 0$

$f''(x) = 4 - 12x^2 = 4(1-3x^2)$

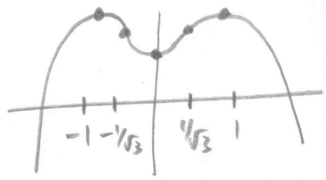
CP: $x = -1, 0, 1$

	$(-\infty, -1/\sqrt{3})$	$(-1/\sqrt{3}, 1/\sqrt{3})$	$(1/\sqrt{3}, \infty)$
$1-\sqrt{3}x$	+	+	−
$1+\sqrt{3}x$	−	+	+
$f''(x)$	−	+	−
$f(x)$	CD	CU	CD

$f(x)$ CU on $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

CD on $\left(-\infty, -\frac{1}{\sqrt{3}}\right) \cup \left(\frac{1}{\sqrt{3}}, \infty\right)$

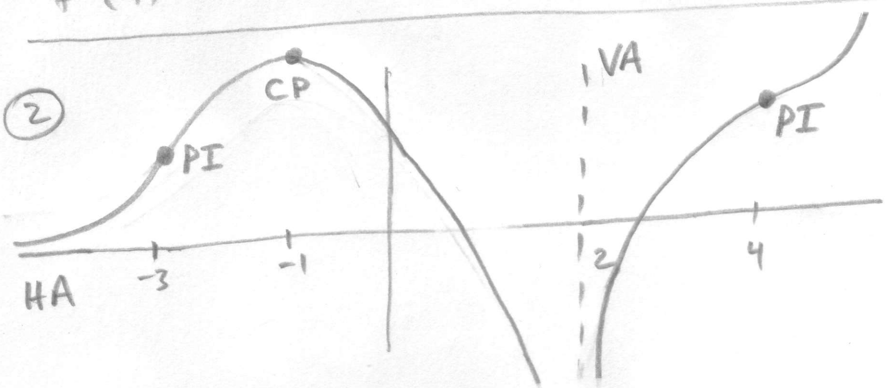
PI $x = -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$



$f''(-1) = -8 < 0 \Rightarrow x = -1$ rel max

$f''(0) = 4 > 0 \Rightarrow x = 0$ rel min

$f''(1) = -8 < 0 \Rightarrow x = 1$ rel max

② 

$y = 0$ HA

$x = 2$ VA

$x = -1$ CP (rel. max)

$x = -3, 4$ PI

③ $V = \pi r^2 h = 16 \Rightarrow h = \frac{16}{\pi r^2}$

$A = 2\pi r^2 + 2\pi r h$

$A(r) = 2\pi r^2 + 2\pi r\left(\frac{16}{\pi r^2}\right) = 2\pi r^2 + \frac{32}{r}$

$A'(r) = 4\pi r - \frac{32}{r^2} = \frac{4}{r^2}(\pi r^3 - 8) \overset{\text{set}}{=} 0 \Rightarrow r^3 = \frac{8}{\pi}$

$A''(r) = 4\pi + \frac{64}{r^3} > 0 \Rightarrow$ So this is rel. min by 2nd derivative test.

$r = 2/\sqrt[3]{\pi}$

$h = 4/\sqrt[3]{\pi}$