Math 329 Exam 2 4/9/2014 Solutions

(1) a) By 3 reflections Theorem, $f = r_a r_b r_c$ ($f = r_a$, $f = r_a r_b$ similar)

Then $f^{-1} = r_c r_b r_a$ isometry since product of reflections, inverse since

$ff^{-1} = (r_a r_b r_c)(r_c r_b r_a) = r_a r_b \cancel{r_c^2}^{\,1} r_b r_a = r_a \cancel{r_b^2}^{\,1} r_a = r_a^2 = 1$

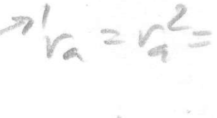
b) Any orient.-preserving isometry of S^2 is a product of rotations (= translations) which we showed is just one rotation. Every rotation of S^2 fixes a pair of antipodal points that lie on the axis of the rotation.

c) Suppose all borders of WY do lie on great circles. By Girard's Thm, the area of a quadrilateral = angle sum $- 2\pi$. Assuming the map preserves angles, then area(WY) $= (4)(\frac{\pi}{2}) - 2\pi = 0$ on S^2, which is impossible.

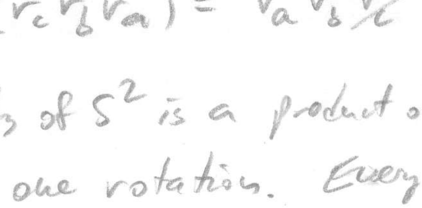
---

(2) a) $f(0,0) = r_4 r_3 r_2 r_1 (0,0) = r_4 r_3 r_2 (0,-2) = r_4 r_3 (-2,0) = r_4 (0,2) = (4,2)$

b) Translation by $\vec{u} = (4,2)$.



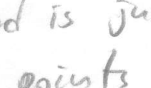
(3) $\ell \perp$ bis of PP', $m \perp$ bis of QQ' $\Rightarrow T = \ell \cap m$



α, β, θ shown

$\theta = \alpha + \beta$

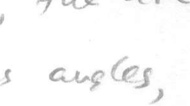
(4) a) Given $\vec{u} \perp \vec{v}$



$d_1 = \vec{u} + \vec{v}$, $d_2 = \vec{u} - \vec{v}$

$|d_1|^2 = (\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v}) = |\vec{u}|^2 + \cancel{2\vec{u}\cdot\vec{v}}^{\,0} + |\vec{v}|^2 = |\vec{u}|^2 - \cancel{2\vec{u}\cdot\vec{v}}^{\,0} + |\vec{v}|^2 = |d_2|^2$

b) Midpoint of $d_1 = \frac{1}{2}(\vec{u}+\vec{v})$. Midpoint of $d_2 = \frac{1}{2}(\vec{u}-\vec{v}) + \vec{v}$
or $\frac{1}{2}(\vec{v}-\vec{u}) + \vec{u}$



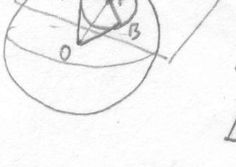
$\frac{1}{2}(\vec{u}+\vec{v}) = \frac{1}{2}(\vec{v}-\vec{u}) + \vec{u} = \frac{1}{2}\vec{u} + \frac{1}{2}\vec{v}$

(5) a) $P \in S^2 \Rightarrow |OP| = 1 \underset{f \text{ isometry}}{\Rightarrow} |f(O)\,f(P)| = 1 \underset{f(0)=0}{\Rightarrow} |O\,f(P)| = 1 \Rightarrow f(P) \in S^2$

b) chordal distance is the distance in $\mathbb{R}^3$, so for any $P, Q \in S^2$,

$|f(P)\,f(Q)|_{chordal} \underset{\substack{f \text{ isometry} \\ \text{of } \mathbb{R}^3}}{=} |PQ|_{chordal} \Rightarrow |f(P)\,f(Q)|_{gc} = |PQ|_{gc} \Rightarrow f$ isometry of S^2

(6)



A, B in $W \cap S \Rightarrow |OA| = 1 = |OB|$, and $AP \perp OP$, $BP \perp OP$ since $OP \perp W$

$\Delta AOP \cong \Delta BOP$ by HL (Pythagorean Thm) since OP is leg of rt. triangles

$\Rightarrow |AP| = |BP|$ corresponding sides $\Rightarrow A, B$ are on circle P.