Math 329 (Geometry) Exam 2

April 9, 2014 Professor Ilya Kofman

NAME: ____

Problem 1.

(a) Prove that every isometry f of $\mathbf{R}^2$ has an inverse f^{-1} which is also an isometry.

(b) Explain why every orientation-preserving isometry of S^2 must have fixed points.

(c) On the map Wyoming appears to be a rectangle. Explain why its borders cannot lie on great circles.

Problem 2. Let $f(x, y) = (r_4 \circ r_3 \circ r_2 \circ r_1)(x, y)$, where

r_1 is the reflection in the line $y = -1$.

r_2 is the reflection in the line $y = x$.

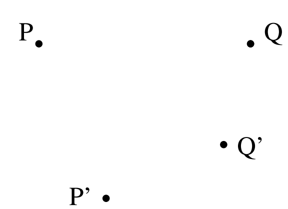
r_3 is the reflection in the line $y = -x$.

r_4 is the reflection in the line $x = 2$.

(a) Calculate $f(0, 0)$.

(b) Given $f(1, -1) = (5, 1)$ and $f(-1, -1) = (3, 1)$, classify the isometry f. Justify.

Problem 3. Rotations $R_{Q,\alpha}$ and $R_{P,\beta}$ satisfy $R_{Q,\alpha}(P) = P'$ and $R_{P,\beta}(Q) = Q'$ as shown. On the diagram below, find T such that $R_{T,\theta} = R_{Q,\alpha}R_{P,\beta}$. Indicate θ, α and β on the diagram, and also express θ using α and β.



Problem 4. Use vectors to prove the following:

(a) The diagonals of a rectangle are congruent.

(b) The diagonals of a parallelogram bisect each other.

Problem 5. Let f be an isometry of $\mathbf{R}^3$ such that $f(0) = 0$.

(a) If P is a point on S^2, prove that $f(P)$ is a point on S^2.

(b) Prove that f is an isometry of S^2.
Hint: We proved that for any pairs of points on S^2,

$$|PQ|_{\text{chordal}} = |P'Q'|_{\text{chordal}} \;\Leftrightarrow\; |PQ|_{\text{great circle}} = |P'Q'|_{\text{great circle}}$$

Problem 6. We proved if a plane W intersects a sphere S in more than one point, then $W \cap S$ is a circle. Let A, B be two points in $W \cap S$. To prove part of this theorem, show that A and B belong to a circle with center P.