Calculus I (Math 231) Exam 2

Date: October 28, 2009 Professor Ilya Kofman

Justify answers and show all work for full credit. No calculators allowed.

NAME: Key

Problem 1 (32pts). *Compute the derivative* $\frac{dy}{dx}$. *Do not simplify. Show all work!*

(a) $y = \dfrac{e^{5x}}{7+\cos(3x)}$

$$y' = \frac{(7+\cos(3x))(5e^{5x}) - (e^{5x})(-3\sin(3x))}{(7+\cos(3x))^2}$$

(b) $y = \left(\sqrt[3]{7x} + \sqrt{x^2+4}\right)^{14}$

$$y' = 14\left(\sqrt[3]{7x} + \sqrt{x^2+4}\right)^{13}\left(\tfrac{7}{3}(7x)^{-2/3} + \tfrac{1}{2}(x^2+4)^{-1/2}(2x)\right)$$

(c) $y = \ln(2 + \tan(3x+4))$

$$y' = \frac{3\sec^2(3x+4)}{2+\tan(3x+4)}$$

(d) $xe^y = y - 1$

$$xe^y \frac{dy}{dx} + e^y = \frac{dy}{dx}$$

$$\frac{dy}{dx}(1 - xe^y) = e^y$$

$$\frac{dy}{dx} = \frac{e^y}{1 - xe^y}$$

Problem 2 (20pts). *Let* $f(x) = \dfrac{1}{2x+3}$.

(a) *Use the definition of the derivative to find* $f'(1)$.

5 $$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{\frac{1}{2(1+h)+3} - \frac{1}{5}}{h} = \lim_{h\to 0} \frac{1}{h}\left(\frac{1}{5+2h} - \frac{1}{5}\right)$$

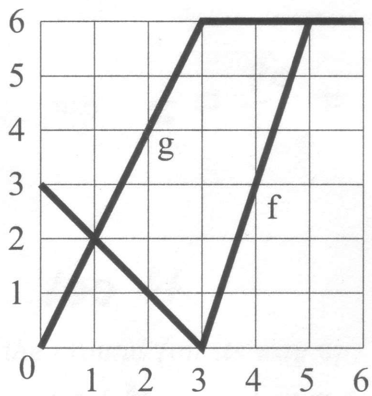
5 $$= \lim_{h\to 0} \frac{1}{h}\left(\frac{5-(5+2h)}{5(5+2h)}\right) = \lim_{h\to 0} \frac{1}{\cancel{h}}\left(\frac{-2\cancel{h}}{25+10h}\right) = \frac{-2}{25}$$

(b) *Use any method to find* $f''(1)$.

2 $$f'(x) = -\frac{2}{(2x+3)^2} = -2(2x+3)^{-2}$$

5 $$f''(x) = 4(2x+3)^{-3}(2) = \frac{8}{(2x+3)^3}$$

3 $$f''(1) = \frac{8}{5^3} = \frac{8}{125}$$



Problem 3 (18pts). *Graphs of* $f(x)$ *and* $g(x)$ *are shown above. Show all work below!*

(a) *Let* $A(x) = f(x)g(x)$. *Find* $A'(2)$.

$$A'(x) = f(x)\,g'(x) + f'(x)\,g(x)$$

2 $$A'(2) = f(2)\,g'(2) + f'(2)\,g(2)$$

4 $$= (1)\cdot(2) + (-1)(4) = -2$$

(b) *Let* $B(x) = f(g(x))$. *Find* $B'(2)$.

$$B'(x) = f'(g(x))\cdot g'(x)$$

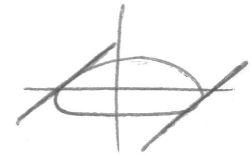
2 $$B'(2) = f'(g(2))\cdot g'(2)$$

4 $$= f'(4)\cdot(2) = (3)(2) = 6$$

(c) *Let* $C(x)$ *be the inverse of* $g(x)$ *for* $0 \le x \le 3$. *Find* $C(2)$ *and* $C'(2)$.

2 $$g(1) = 2 \Rightarrow C(2) = 1$$

4 $$C'(2) = \frac{1}{g'(1)} = \frac{1}{2}$$



Problem 4 (10pts). *Find the two points on the ellipse $x^2 + 2y^2 = 1$ where the tangent line has slope $= 1$.*

$$x^2+2y^2=1, \quad \frac{dy}{dx}=1$$

6 $$2x+4y\frac{dy}{dx}=0 \Rightarrow 2x+4y(1)=0 \Rightarrow x=-2y$$

2 $$\Rightarrow (-2y)^2+2y^2=1 \Rightarrow 4y^2+2y^2=1 \Rightarrow y^2=\frac{1}{6}$$

2 $$\Rightarrow y=\pm\frac{1}{\sqrt{6}} \quad x=-2y \Rightarrow \left(-\frac{2}{\sqrt{6}},\frac{1}{\sqrt{6}}\right) \text{ and } \left(\frac{2}{\sqrt{6}},-\frac{1}{\sqrt{6}}\right)$$

Problem 5 (12pts). *A ball is thrown up from the ground with a velocity of 80 ft/sec.*

(a) *Find the maximum height of the ball.*

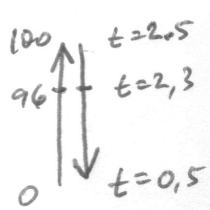
$$s(t)=80t-16t^2$$

3 $$v(t)=s'(t)=80-32t \overset{set}{=} 0 \Rightarrow t=\frac{80}{32}=\frac{5}{2}=2.5\text{ sec}$$

$$s(2.5)=(80)\left(\frac{5}{2}\right)-16\left(\frac{25}{4}\right)$$

3 $$=200-100=100\text{ ft.}$$

(b) *Find the velocity of the ball when it is 96 feet above the ground (on its way up).*



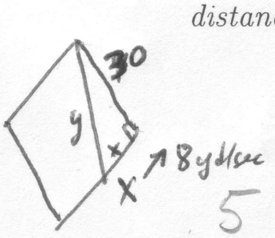
$$s(t)=80t-16t^2=96 \Rightarrow 16t^2-80t+96=0$$

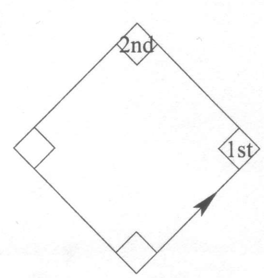
$$t^2-5t+6=0$$

4 $$(t-2)(t-3)=0 \Rightarrow t=2 \text{ or } 3\text{ sec.}$$

2 $$v(2)=80-32(2)=80-64=16\text{ ft/sec}$$

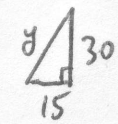
Problem 6 (15pts). *A baseball diamond is a square with sides 30 yards. A batter hits the ball and runs toward first base with a speed of 8 yd/sec. At what rate is his distance to second base decreasing when he is halfway to first base?*





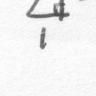
$$x^2+30^2=y^2, \quad \frac{dx}{dt}=-8, \quad x=15$$

5 $$2x\frac{dx}{dt}=2y\frac{dy}{dt}$$



5 $$x=15 \Rightarrow y=15\sqrt{5}$$

$$\cancel{(2)}\cancel{(15)}(-8)=\cancel{(2)}(\cancel{15}\sqrt{5})\frac{dy}{dt}$$



5 $$\frac{dy}{dt}=-8/\sqrt{5} \approx -3.6\text{ yd/sec}$$