

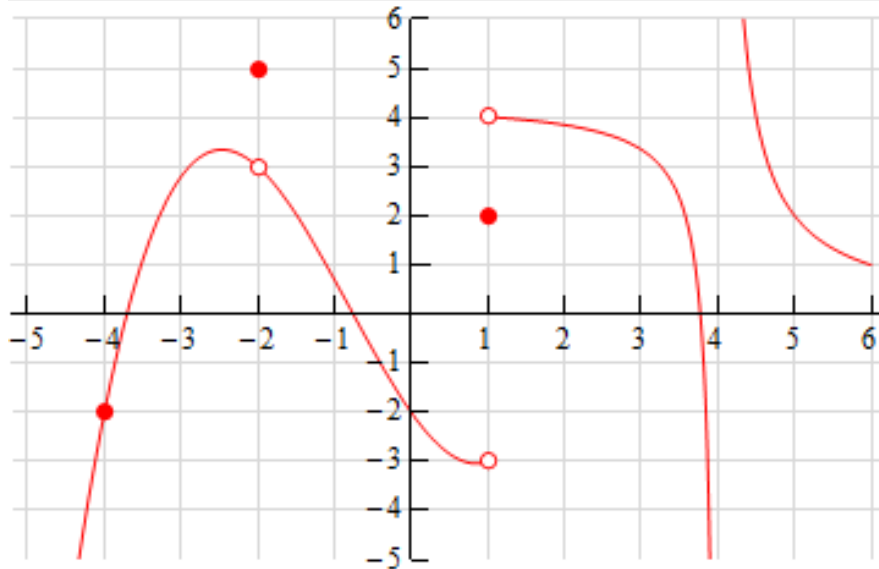
Business Calculus I (Math 221) Exam 1

September 28, 2016

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Justify answers and show all work for full credit. No calculators permitted on this exam.

NAME: _____



Problem 1 (20pts). The graph of $y = f(x)$ is shown above. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary for this problem.

(a) $\lim_{x \rightarrow -2} f(x) =$

(b) $\lim_{x \rightarrow -4} f(x) =$

(c) $\lim_{x \rightarrow 1^+} f(x) =$

(d) $\lim_{x \rightarrow 1^-} f(x) =$

(e) $\lim_{x \rightarrow 1} f(x) =$

(f) $\lim_{x \rightarrow 4^-} f(x) =$

(g) For $f(x)$ to be continuous at $x = -2$, we must set $f(-2) =$

(h) Estimate the derivative $f'(-1) =$

(i) Estimate the derivative $f'(-3) =$

(j) Estimate for which x the derivative $f'(x) = 0$, $x =$

Problem 2 (12pts). Evaluate these limits. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify. **Show all work!**

(a) $\lim_{x \rightarrow 4} \frac{x^2 - 13x + 36}{x - 4}$

(b) $\lim_{x \rightarrow -3} \frac{x^2 - 4x - 21}{x^2 - 9}$

(c) $\lim_{x \rightarrow 5^-} \frac{1}{2x - 10}$

(d) $\lim_{x \rightarrow \infty} \frac{6x^5 + 8x^3 - 1}{-7x^5 + 3x^4 - 2x}$

Problem 3 (8pts). Recall $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$.

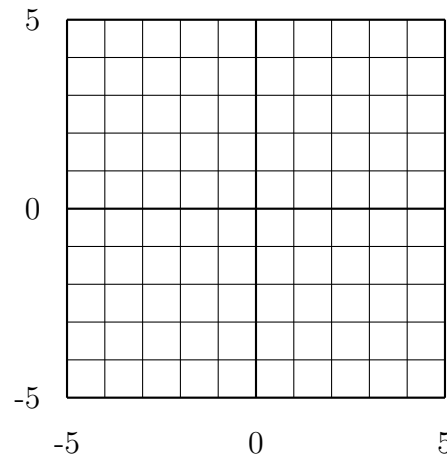
(a) If $f(x) = 4x^3$, write the limit for $f'(2)$. Do not evaluate this limit.

(b) Show that $g(x) = |x|$ is not differentiable at 0. Evaluate this limit. **Show all work!**

Problem 4 (5pts). (a) On the grid below, graph the following piecewise defined function.

$$f(x) = \begin{cases} 3 - 2x & x < 3 \\ x - 4 & x \geq 3 \end{cases}$$

(b) Is the function $f(x)$ continuous at $x = 3$? (Do not justify.) **YES** **NO**



Problem 5 (6pts). For what value of c (if any) is the function $g(x)$ continuous at $x = 4$? Justify your answer.

$$g(x) = \begin{cases} \frac{x^2 + 2}{2x - 2} & x < 4 \\ c & x = 4 \\ x^2 - 3x - 1 & x > 4 \end{cases}$$

Problem 6 (24pts). Compute the derivative $y' = \frac{dy}{dx}$. Do not simplify. Show all work!

(a) $y = \frac{x^4}{3} + 8x^{3/4} - 5x + 7 + 15x^{-1/5}$

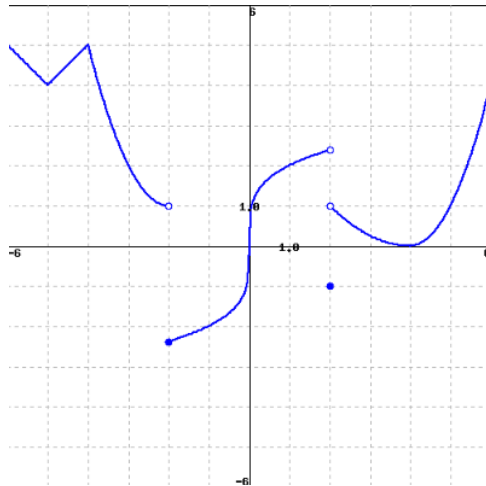
(b) $y = \frac{2}{\sqrt[3]{x}} - 4\sqrt{x^7} + \frac{6}{x} + \frac{3}{x^5}$

(c) $y = \sqrt[3]{3x^4 - 2x^3 - 4}$

(d) $y = \frac{6x^5 + 4x^3}{x^8 - 4}$

(e) $y = (4x^5 + 3x^4 + 20)(4x^9 - 10)$

(f) $y = \sqrt{(3x - 4)^5 - 10x}$



Problem 7 (8pts). The graph of $y = f(x)$ is shown above for $-6 < x < 6$.

(a) For which x values is $f(x)$ not continuous?

(b) For which x values is $f(x)$ not differentiable?

(c) For which x values is the derivative $f'(x) = 0$?

Problem 8 (7pts). Let $F(x) = 4x^3 - 2x^2 - 12$. Find the equation of the tangent line to the graph of $F(x)$ at $x = 1$. Leave your answer in the form $y = mx + b$.

Problem 9 (8pts). Let $g(x) = (2x - 1)^5$.

(a) Find $g'(0)$.

(b) Find $g''(0)$.

Problem 10 (12pts). For x units sold, the total revenue function is $R(x) = 30x + 200$. The total cost function is $C(x) = 600 + 9x + \frac{1}{8}x^2$.

(a) Find the profit function $P(x)$.

(b) Find the marginal profit when 100 units are sold.

(c) If $P(100) = 450$, use your part (b) answer to estimate the total profit if 101 units sold.

(d) Should the company sell the 101st unit? Explain using your answers above.